Deposition of Solid Particles on a Collector: Formulation of a New Theory

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Two concepts concerning the deposition of particles from suspension onto a collector, the shadow effect due to deposited particles and the singular and random behavior of approaching particles, are presented and discussed. Based on these concepts, a new theory of particle deposition is proposed and procedures for the simulation of deposition process outlined. The simulation results give the rate of deposition and the deposit morphology. Potential applications of this model to a number of technical problems are outlined.

SCOPE

Flow of a fluid-particle suspension past a solid boundary often results in the deposition of particles on the boundary surface. One method of estimating the rate of deposition is the classical trajectory analysis developed by Sell (1931) more than 40 yr ago. Through the determination of particle trajectories from the solution of the appropriate equations of particle motion, a limiting trajectory which separates approaching particles which will deposit on the boundary from those which will not can be located, which, in turn, enables the estimation of the rate of deposition. The classical trajectory analysis does not consider the effect of deposited solid particles on subsequent deposition and hence does not give information about the progression of the deposition process. It is therefore only useful in obtaining macroscopic information during the initial period of deposition.

In this paper, we present two concepts which are characteristic of particles in suspension in terms of their interaction with a collector surface, namely, the shadow effect created by the deposited particles and the singular and random behavior of approaching particles. Deposition process can be considered in terms of the interplay of these two concepts. The random distribution of approaching particles in their upstream positions imparts a stochastic feature to the process. However, the determination of the particle trajectories is based on the solution of the equations of particle motion, which is deterministic.

It is shown that based on these two concepts, a new model of particle deposition may be constructed. The ultimate aim is the formulation of a comprehensive deposition model, which can be used for the study of certain phenomena associated with particle deposition, as a basis of optimum filtration design.

CONCLUSIONS AND SIGNIFICANCE

The theory of particle deposition presented in this work is formulated on the basis of the interplay of two concepts which are characteristic of particles in a suspension flowing past a collector. It is therefore capable of predicting, in a detailed manner, the progressions of the deposition process.

As shown in an illustrative example, with the use of this theory detailed information on the deposition process can be predicted in a stepwise manner, which includes not only the rate of deposition but also the morphology of the deposits. The theory makes it possible to study, in a rigorous manner, a number of practically important problems associated with filtration, such as pressure drop increase during filtration, filter cleaning, etc., the study of which, up to the present, has been handicapped because of the lack of a proper theoretical framework.

When a fluid-particle suspension flows past a solid boundary, particle deposition may take place along the boundary under the influence of a number of forces. This phenomenon which often occurs in natural and man-made environments as well as in a number of technical applications has been a subject of active study for several decades. The principal aspect of these studies has been concerned with the prediction of the deposition flux. Sell (1931), in a pioneer paper more than 40 yr ago, calculated the particle deposition due to inertial impaction from suspensions to collectors of different shapes. His approach has since been adopted and refined by a number of workers on aerosols (Albrecht, 1931; Langmuir and Blodgett, 1944; Bosanquet, 1950) and hydrosol (Yao et al., 1971; Spielman and Fitzpatrick, 1973; Payatakes et al., 1974; Rajagopalan and Tien, 1976) and forms the basis of what is known today as trajectory analysis.

The basic idea of trajectory analysis and its use in estimating particle deposition rate is simple and straightforward. Based on the knowledge of the flow field around a collector and by taking into account all the forces acting

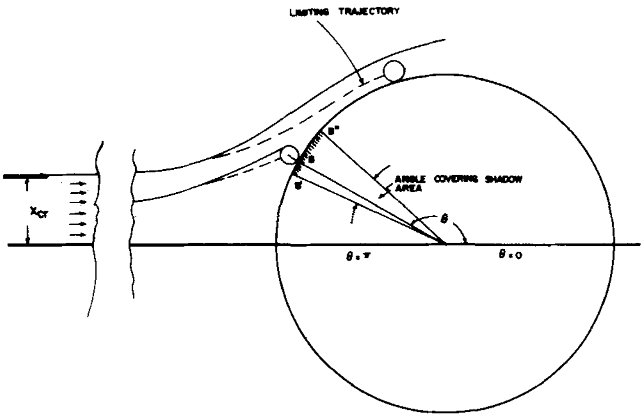

Fig. 1. Particle deposition on single collector.

on a particle, the trajectory of that particle can be determined from the equations of motion. If a criterion of particle attachment upon its contact with the collector is assumed (the simplest one would be that particles attach to the surface upon contact), it would be possible to determine whether or not a particle beginning at a given position would be captured by the collector as it moves downstream. With this knowledge, the prediction of the rate of particle deposition becomes a simple matter.

An illustration of the trajectory concept is shown in Figure 1 in which a spherical collector is placed in a flowing suspension. The solid line around the collector represent streamlines and the dotted lines particle trajectories. To calculate the total amount of deposition or the deposition flux, it is only necessary to determine the so-called limiting trajectory which separates the particles that would make contact with the collector from those that would not. The limiting trajectory is defined as the trajectory along which a particle barely makes contact with the collector. Let X_{cr} be the distance between the limiting trajectory and the central axis upstream; the rate of collection over the entire collector surface is

$$W = (\pi X_{cr}^2)(V)c \tag{1}$$

Conventionally, a collection efficiency η is defined as the ratio of the particles collected to those passing through the projected area of the collector upstream. For a spherical collector, η is given as

$$\eta = \left(\frac{X_{cr}}{a_c}\right)^2 \tag{2}$$

Theoretical investigations on particle deposition in the past have been made within the framework of the trajectory analysis. As valuable as it is, trajectory calculation can provide only partial answers to the problem of particle deposition. For example, in fibrous filtration of solid aerosol particles, it has been observed that with increased accumulation of particles, porosity and permeability may decrease, pressure drop may increase, particle penetration may decrease, and accumulated deposited particles apparently form chainlike branched trees or dendrites at least during the initial step of buildup (see Billings, 1966; Davies, 1973). These phenomena certainly cannot be predicted from the trajectory calculation alone.

A more basic examination on the application and the limitation of the trajectory analysis in the study of particle deposition can be made along the following lines. Since deposition is principally a process of interaction between particles in suspension and collector surface, it is essential that in the study of particle deposition, the collector surface should be represented in the same way as it is being seen by the particles. The deposition of particles on a collector surface changes the surface characteristics. This fact has come to the attention of previous investigators as evidenced by the facts that the collection efficiency calculated from the trajectory analysis is often referred to as clean collector efficiency or initial collection efficiency and that phenomenological approaches have been used to predict the effects of particle accumulation (Billings, 1966).* There exists no generalized predictive method which accounts for the effect of deposition on collection efficiency.

It would appear that for a more complete analysis of the particle deposition phenomenon, the effects due to particle deposits should be included. It is necessary not only to determine the rate of particle deposition, but also the structure and geometry of the deposited particles. For such a study, additional insights concerning the interaction between the particles and collector and the manner in which these interactions manifest themselves are required. New concepts and assumptions underlying the interaction process have to be established. The formulation of such concepts and their utilization in the study of particle deposition represent the major purpose of this work.

BASIC CONCEPTS

The two basic concepts in the study of particle deposition are the finiteness of particle size and the manner in which approaching particles in a flowing suspension move toward the collector. The essential feature of these two concepts and the interaction between them in the deposition process were described briefly in a recent communication by the present authors (Tien, Wang, and Barot, 1977). In the following, we shall present a more detailed description of these concepts in general terms and illustrate them by a two-dimensional problem.

Shadow Effect†

The importance of a particle possessing a finite size in particle deposition has long been recognized and is referred to as the interception effect. Its recognition is limited to the calculation of the rate of deposition. Equally important is the fact that once a particle is deposited, it creates a shadow area around itself on the collector surface, within which no subsequent particle deposition may take place. This is illustrated by arc B′BB″ for the deposited particle A shown in Figure 1 for the case of inertial impaction and interception. It is to be noted that the creation of a shadow area is a result of finite size of particles and will take place in any force field.

The creation of shadow areas by deposited particles has two consequences. Since there will be no deposition within any shadow area, it means that particle collection takes place at discrete positions along a collector surface. If the shadow areas represent a significant part of the total collector surface, the deposited matter cannot be in the form of a smooth coating.

The second consequence arises from the fact that with

* It should be noted that a similar empirical method was employed earlier in the study of the dynamic behavior of deep-bed filter for liquid systems (see Ives, 1969).

† The term shadow effect was used previously by Clarenburg and Werner (1965) to describe the possible interference between two neighboring fibers in an aerosol filter bed. In the present work, shadow effect is used to denote the effect of deposited particles on subsequent particle deposition.

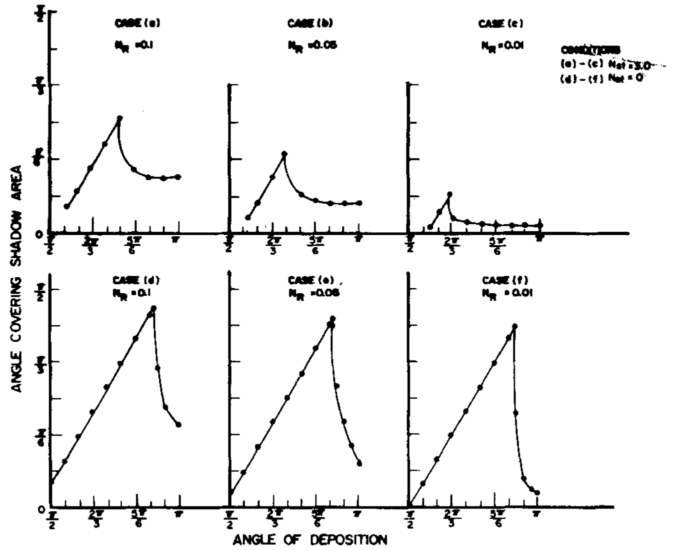

Fig. 2. Shadow area vs. angle of deposition.

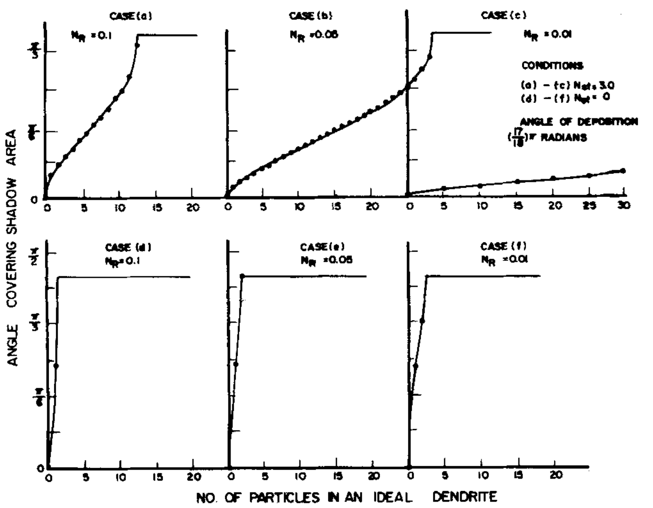

Fig. 3. Shadow area as a function of dendrite size.

the creation of shadow area, subsequent approaching particles which would have deposited within the shadow area had there been no deposition now attach themselves to the deposited particle. This results in the formation and growth of chainlike particle tree or dendrites as observed in earlier experiments (Billings, 1966).

The magnitude of the shadow area due to a deposited particle is a function of the location of the deposited particle and other variables effecting particle trajectories. Its determination can be made from the knowledge of the particle trajectories through trial and error. Assuming that particle A becomes deposited at point B on the collector, by tracing out trajectories with initial positions in the neighborhood of that of A and by locating the two trajectories which barely miss the deposited particle from either above or below, arc B′B″ can be estimated as shown in Figure 1.

As an example, calculations were made for a spherical collector using Stokes flow. In Figure 2, the shadow area, expressed in terms of the angle it covers, created by the deposition of a particle on a spherical collector is given as a function of the position of deposition for three different values of the relative size parameter (or traditionally known as interception parameter) (N_R) and two different values of the Stokes number (N_{St}). The forward stagnation point is at π rad (or 180 deg),

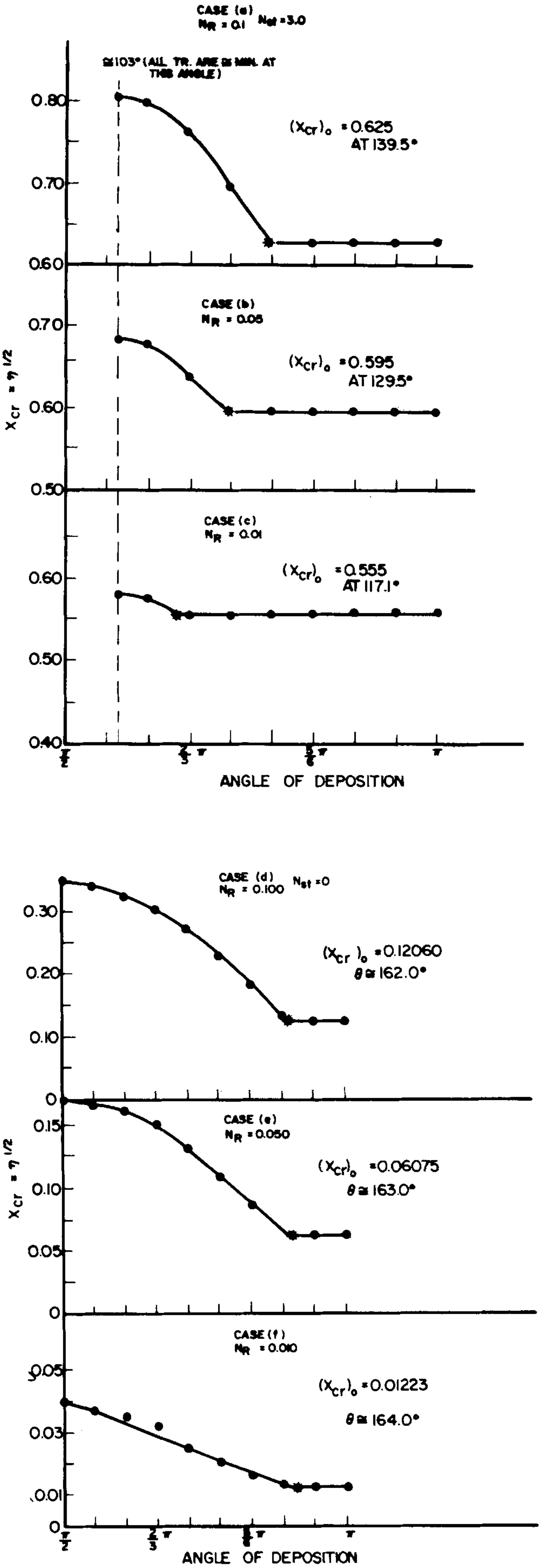

Fig. 4. Collection efficiency increase vs. deposition angle for a spherical collector with one deposited particle.

and it is shown that the shadow area increases as the position of deposition moves away from the forward stagnation point. A maximum is reached when the upper edge of the shadow (that is, point B″ in Figure 1) coincides with the limiting angle of deposition which is at $\pi/2$ (90 deg) for the case $N_{St} = 0$ and at approximately 100 deg for $N_{St} = 3.0$. The shadow area increases with the increase of the particle size (or the relative size parameter). But the most important factor affecting the shadow area appears to be the Stokes number. At $N_{St} = 0$, even at relatively small values of N_R, the shadow area is fairly significant.

The concept of shadow area is also true for particles which become attached to deposited particles. Shadow area created by an ideal dendrite* is calculated as a function of the size of the dendrite for a fixed deposition position. This is shown in Figure 3. Because of its orientation, this increase is limited only in the upper part of the shadow area (that is, arc BB″ in Figure 1). Again, the effect due to the Stokes number is significant. For example, for $N_R = 0.05$, at a deposition angle of 170 deg, an ideal dendrite with two particles will blank the entire upper part of the collector at $N_{St} = 0$. Almost thirty particles are required to do the same if $N_{St} = 3.0$.

Another consequence of the particle deposition is the increase of the collection efficiency. With the presence of deposited particles, the collection system (collector plus deposited particle) may capture particles from the suspension, which would escape from a clean collector. This increase in η can be represented by the increase of the lateral distance of the limiting trajectory from the central axis x_{cr}. For the simple case of a collector with one deposited particle, x_{cr} is given as a function of the deposition angle with specified values of N_R and N_{St} as shown in Figure 4.

It should be pointed out that the results shown above (Figures 2 to 4) are based on a two-dimensional flow consideration. The limit of the shadow effect along the azimath angle direction therefore cannot be displayed. As such it presents a distorted view about the shadow areas effect. Furthermore, in determining particle trajectories, only the inertial and intercept effect were considered, and the possible change of the flow field due to the presence of deposited particles was ignored. These results therefore should not be used for quantitative purposes.

Singular and Random Behavior of Approaching Particles

Even if particle concentration throughout the suspension is assumed to be uniform macroscopically, the locations of the individual particles in the fluid at any instant do not follow any regular pattern. At large distance upstream from the collector, particles may be assumed to move along streamlines, but their travel patterns are not ordered. For particles approaching the collector far away from it, they are positioned along streamlines at irregular intervals. Accordingly, if an imaginary control surface is defined to be part of a plane normal to the direction of the main flow which comprises the projected area of the collector on the plane (either in its entirety or partially), the passing of the particles across the control surface can be considered to be singular† (that is, one particle at a time), and the positions of the particles in the control surface upon their arrival are randomly distributed (that is, the probability of a particle arriving at a given point within the control area is the same for all points within the control area).

This characterization of approaching particle behavior, singular and random, may be taken as basic assumptions. The randomness behavior can be inferred from the fact that particle concentration is dilute [for example, for a concentration of 10^6 particle (1 μm in diameter)/cm^3 of air, the volume fraction occupied by particles is less than one millionth]. The assumption of particles arriving in singles is true if the spatial distribution of particles in the suspension obeys the Poisson distribution, a condition which is known to be valid for aerosols (Green, 1927). Alternatively, its validity may be argued intuitively on physical grounds. Let A be the area of the control surface. For a time interval of Δt, the number of particles passing through the control surface N is*

$$N = (V)(A)(c)(\Delta t) \tag{3}$$

Thus, it is always possible to select a Δt such that N is equal to one.

The phenomenon of particle deposition therefore can be considered in terms of the singular and random behavior of the approaching particles and the effect of the shadow areas created by the deposited particles. Formulation of the particle deposition can be made accordingly.

SIMULATION OF PARTICLE DEPOSITION

To simulate a given particle deposition process, the following general procedure can be used. A control surface is selected, which is placed sufficiently upstream where particles can be assumed to follow the streamlines of the fluid. The positions of arrival of succeeding particles on the control surface can be determined by a sequence of random numbers, each of which corresponds to a particular position on the control surface. With the knowledge of the initial positions of approaching particles, the flow field around the collection system, and the kind of forces involved in the particular problem, the trajectories of the approaching particles in the order of their arrival can be determined. With an assumed criterion of particle attachment, the outcome of each approaching particle (that is, whether it will escape, deposit upon the collector or deposit upon particles already deposited) can be easily determined. The main feature of this model can be summarized as follows:

1. The model is both stochastic and deterministic. The stochastic nature of the model derives directly from the randomness of the positions of the approaching particles at the control surface. However, with the knowledge of its initial position and the flow field around the collector and the operating forces, the trajectory of a given approaching particle is specified.
2. Because of the stochastic nature of the model, the ensemble average of a large number of simulation gives the predicted behavior of the deposition process.
3. Since the outcome of the approaching particles is determined in the order of their arrival, it is implicitly assumed that the deposition of the late arriving particles does not affect the outcome of the earlier arriving particles.
4. The control surface should be large enough to accommodate all particles which will be captured by the

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* An ideal dendrite is defined here as one in which the centers of all particles forming the dendrite are on a straight line passing through the center of the collector.

† This singular behavior does not imply that the interval between succeeding arrivals is constant.

* A particle is considered to be passing through the control surface only if its center is within the boundary of the surface at the time of crossing. The selection of control surfaces will be discussed in the next section.

collector. Since collection efficiency increases as particle deposition proceeds, simulation should stop when the control surface no longer contains all the approaching particles which will be captured by the collection system (collector plus its deposits).

5. As a linkage between the present model and the trajectory analysis, consider the following. Let M be the number of particles which have been considered in the simulation. Among the M particles, m become captured. Accordingly, an efficiency E can be defined as

$$E = \frac{m}{M} \tag{4}$$

M, m and E are all functions of time. Let t be the time over which M particles have been considered. By the definition of the collection efficiency, for a spherical collector with radius a_c, m is given as

$$m = (\pi a_c^2)(V)(c)\int_o^t \eta dt \tag{5}$$

Similarly M, the number of particles considered in the simulation, represents the number of particles passing through the control surface for $t = 0$ to $t = t$, or

$$M = (A)(V)(c)(t) \tag{6}$$

Combining Equations (4), (5), and (6), we get

$$E = \frac{\pi a_c^2}{A}\frac{\int_o^t \eta dt}{t} = \left(\frac{\pi a_c^2}{A}\right) n_o \frac{\int_o^M \left(\frac{\eta}{\eta_o}\right) di}{M} \tag{7}$$

Where η_o is the clean collector efficiency, Equation (6) can be used to test the consistency of the simulation results.

To illustrate the principle presented above, the problem of deposition on a spherical collector is considered. The flow field around the collector is assumed to be that of creeping flow. The other conditions are

$$N_R = \frac{a_p}{a_c} = 0.1$$

$$N_{St} = 0$$

The condition of $N_{St} = 0$ implies that particle trajectories coincide with streamlines and the mechanism of particle deposition is by interception only. If the effect due to the presence of deposited particles on the flow field is ignored, the particle trajectory expression is given as

$$r^2\left(1 - \frac{3}{2r} + \frac{1}{3r^3}\right)\sin^2\theta = X_o^2$$

That is, the coordinate of the trajectory (r, θ) is determined by the distance from the location of particles on the control surface to the center axis X_o.

It should be emphasized that these assumption (collection by interception only and no effect on flow field due to deposited particles) are not central to the concepts presented in the work, and the formulation of the theory is not contingent upon the use of these assumptions. They are used in this illustration example as a matter of convenience.

For simplicity, the problem of deposition is assumed to be two dimensional. As a result of this assumption, a particle approaching a dendrite on a collector can only collide with it or go over it but not bypass it from the sides. The control surface is arbitrarily placed at a distance of thirty times the collector radius away from the collector (upstream). At this distance, all the streamlines

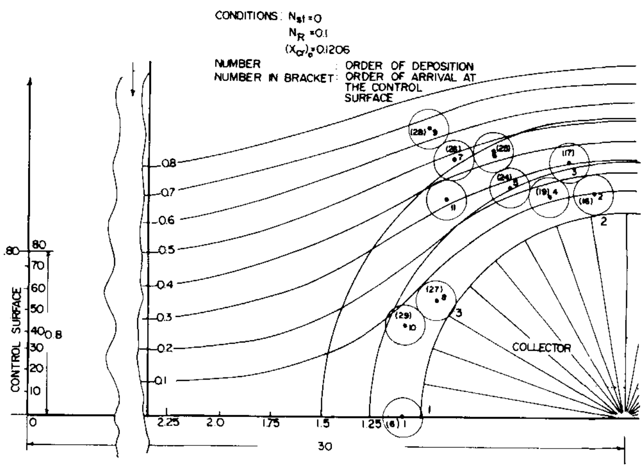

Fig. 5. Simulation results.

are practically parallel to one another, and particles follow streamlines (this is true whether N_{St} vanishes or not).

Because of the two-dimensional assumption, the control surface reduces to a control line extending away from the axis of symmetry and is taken to be $0 < X < 0.8$, where X is the lateral distance from the axis (nondimensionalized by the collector radius). For this example, the limiting trajectory for the clean collector is located at

TABLE 1. SIMULATION RESULTS

Creeping flow around spherical collector $N_R = 0.1$, $\psi = 0$
Control surface taken at $Z = -30$, $0 < X < 0.8$
Initial collection efficiency $\eta = 0.0145$

Particle considered	Initial position	Type of collection*	Dendrite† formed	η	E
1	41	N	no	0.0145	0
2	26	N	no	0.0145	0
3	44	N	no	0.0145	0
4	79	N	no	0.0145	0
5	49	N	no	0.0145	0
6	0	P	[1]	0.0145	0.1667
7	79	N	[1]	0.0145	0.1444
8	42	N	[1]	0.0145	0.125
9	52	N	[1]	0.0145	0.1111
10	50	N	[1]	0.0145	0.1000
11	15	N	[1]	0.0145	0.0909
12	23	N	[1]	0.0145	0.0833
13	13	N	[1]	0.0145	0.0769
14	35	N	[1]	0.0145	0.0714
15	75	N	[1]	0.0145	0.0667
16	11	P	[1,1]	0.1156	0.125
17	32	S	[1,2]	0.2704	0.1765
18	70	N	[1,2]	0.2704	0.1111
19	15	S	[1,3]	0.2704	0.2105
20	61	N	[1,3]	0.2704	0.2000
21	64	N	[1,3]	0.2704	0.1905
22	70	N	[1,3]	0.2704	0.1818
23	66	N	[1,3]	0.2704	0.1739
24	26	S	[1,4]	0.2704	0.2083
25	46	S	[1,5]	0.4160	0.2400
26	48	S	[1,6]	0.4365	0.2693
27	6	P	[1,6,1]	0.4356	0.2963
28	65	S	[1,7,1]	0.4365	0.3214

* N means no collection, P means collection directly onto collector, S means collection by deposited particles.
† Number of aggregates found and their individual size, that is, [3,1,2] means that there are three formed $[a,b,c]$ in the order of $a,b,$ and c, and the first consists of three particles, the second of one particle, and the third of two.

$X = 0.1206$. With the selection of $0.0 < X < 0.8$ as the control line, the simulations can be continued until $\eta/\eta_o = (0.8/0.1206)^2 = 44.44$.

To simulate the random positions of approaching particles at the control line, the line of $0 < X < 0.8$ is divided into eighty segments, with an interval of 0.01, and the nodal points are numbered consequently from 0 to 80 as shown in Figure 5. A sequence of random numbers $[(y_i)$ with max $y_i = 80]$ is generated and represents the initial position of the succeeding approaching particles; using the criterion that particle attachment occurs upon contact, the fate of each particle can be determined.

An example of simulation and the summary of its results are given in Table 1. Column 1 of the table lists the order of arrival of the approaching particles in consecutive numbers. Column 2 gives the position of the approaching particles determined by the random number sequence, each corresponding to a nodal point in Figure 5. For example, the trajectory of the first particle beginning at position 41 ($X = 0.41$) did not intersect the outline of the collector as did the four succeeding particles. The sixth particle starting at position zero deposited at the forward stagnation point. Particle sixteen with its initial position at point 11 touched the collector at an angular position of approximately 98 deg. The next particle starting at position 32 was captured by the preceding deposited particle. It is interesting to compare the outcome of particle 17 with that of particle 2. The trajectory of particle 2 beginning at position 26, which was closer to the center axis than that of particle 18, did not intersect, while the latter trajectory intersected a previously deposited particle. This demonstrates the importance of the effect of deposited particles on the increase of collection efficiency. Particle 19 beginning at position 15 was found to attached itself to particle 17 in its deposited position as a consequence of the shadow effect. Column 3 describes the outcome of each approaching particle passing through the control surface with the use of *P*, *S*, and *N* designating primary collection (deposition on collector), secondary collection (deposition on deposited particles), and no collection. The morphology of the deposits is represented by a matrix, the members of which give the size of the particle dendrite (in terms of particles) in their order of formation. This is shown in column 4. For example (3 1 2) means that there are three dendrites formed with three particles in the one formed first, one particle in the one formed second, and two particles in the dendrite formed last. The collection efficiency η is given in column 5. Note that the increase in η is discrete, and η is calculated by determining the location of the limiting trajectory, taking into account the presence of dendrite. In column 6, the efficiency E defined by Equation (4) is given. This simulation lasts

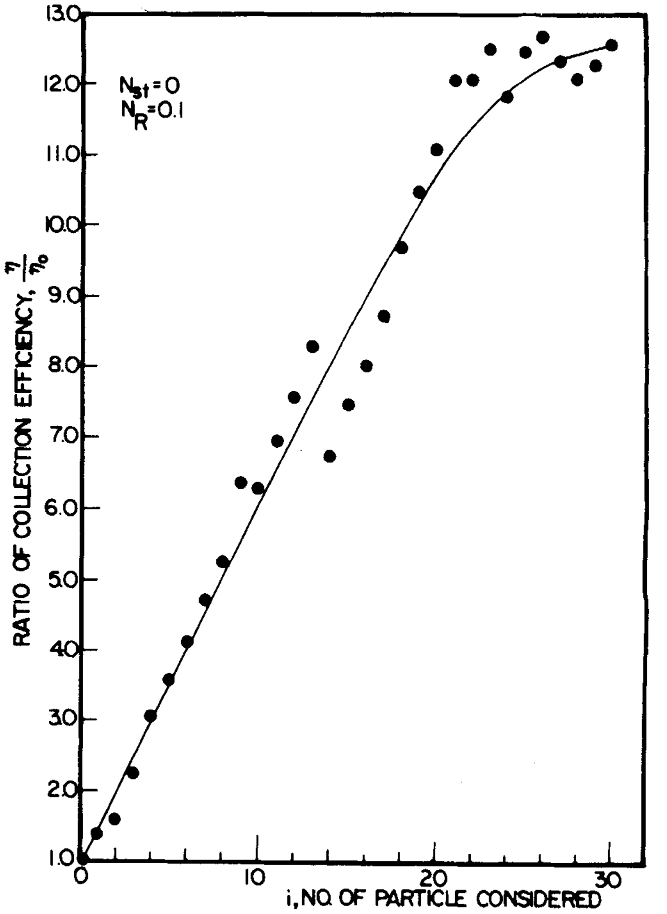

Fig. 7. Change of collection efficiency.

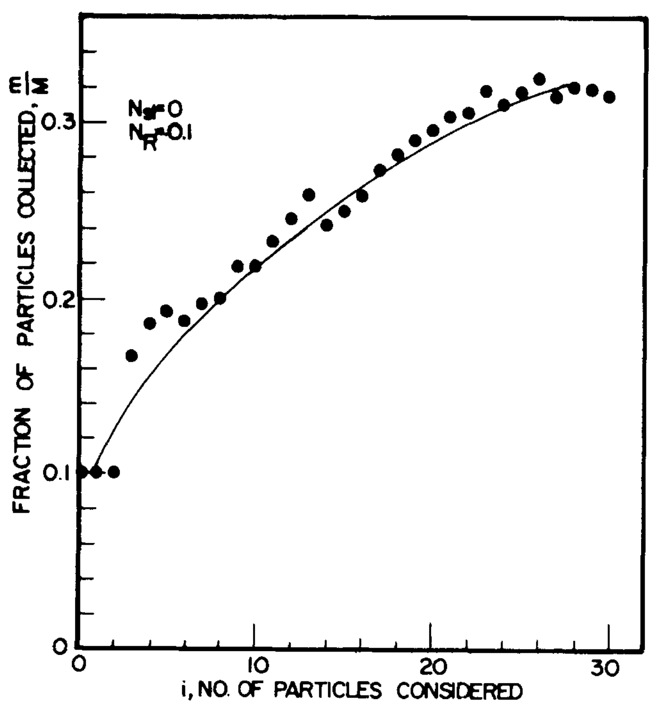

Fig. 6. Fraction of particles collected as a function of number of particles considered for a control surface $0 < X < 0.8$.

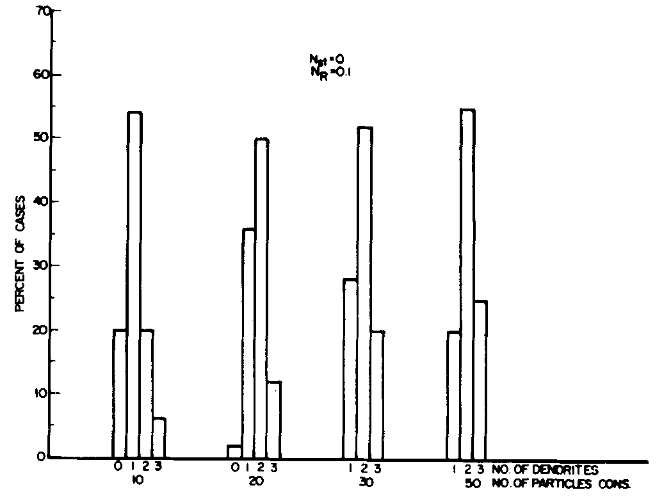

Fig. 8. Classification of simulation runs according to the number of dendrites formed.

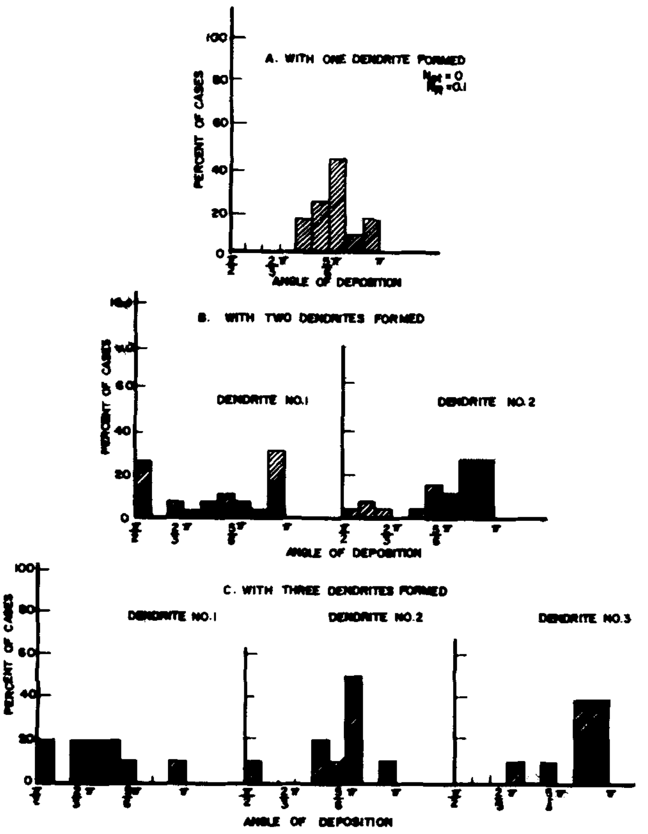

Fig. 9. Simulated results on the position of dendrites.

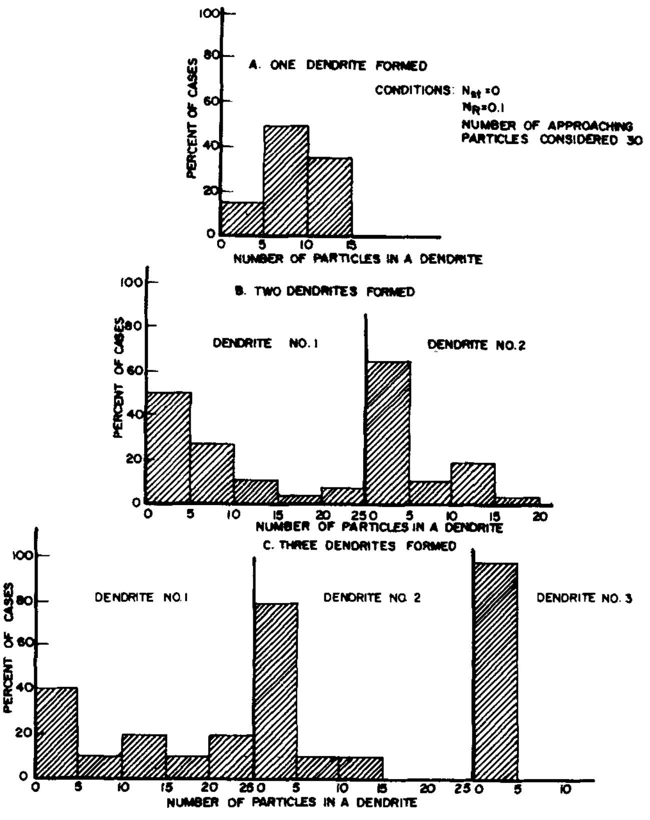

Fig. 10. Size distribution of dendrites formed.

until the 28th particle, when the collection efficiency exceeds $(0.8)^2 = 0.64$. The morphology of the deposits for this simulation is shown in Figure 5.

Because of the stochastic nature of the model, the results obtained from a single simulation such as those given above represent just one of the infinite number of possibilities of the problem. To obtain more conclusive information about the deposition phenomenon, a large number of simulations under the same conditions have to be made, from which an average situation can be obtained. This was done for the example problem by making the same simulation fifty times each with a different sequence of random numbers* to represent the arrival position of the approaching particles.

The average results obtained from the fifty simulations can be grouped into two categories, those pertaining to the rate of particle deposition and those relating to the morphology of the deposits. For the first kind, the collection average efficiency E defined by Equation (4) is given as a function of M [note that M, the number of particles considered, is related to time by Equation (5)] as shown in Figure 6. The collection efficiency η is given as a function of M and is shown in Figure 7.

It is important to note that the results of Figures 6 and 7 which were obtained from two-dimensional simulation do not obey the relationship of Equation (7). This arises from the fact that the method used in designating positions for approaching particles along the control line segment, if generalized to a three-dimensional case, means that the position of approaching particles on the control surface (which in this case is a circle) is to be made on an equal radial distance basis. This means that the points are more concentrated near the center. The condition of uniform particle concentration macroscopically is therefore not satisfied.

The lack of regularity in deposition patterns necessitates the use of a statistical means to describe the simulation results. For the purpose of illustration, consider the following two kinds of information which reflect strongly on the deposition phenomenon: the number of the dendrites formed and their position. The results of the fifty simulations show that for $N_R = 0.1$, the number of dendrites formed varies from one to three. The histograms representing the percentage of each case are shown in Figure 8, with the number of particles considered ranging from ten to fifty.

Figure 9 gives the data on dendrite positions based on the consideration of thirty approaching particles. For those cases where only one dendrite was formed, the position of dendrite varied from 130 to 180 deg, with the plurality of cases between 150 to 160 deg. When more than one dendrite was found, there did not appear to be any preference regarding locations of dendrite formation. The size of a dendrite can be measured in terms of the number of particles it contains. This information is given in Figure 10. For cases with only one dendrite, most of them (more than 80% of the cases) had between five and fifteen particles. For cases when more than one dendrite was found, the majority of dendrites had less than five particles. On the other hand, dendrites as large as having twenty-five particles were observed in cases where three dendrites were formed. These were invariably the cases where efficient collection occurred.

Another way of expressing the dendrite size is to measure its height (in terms of collector radius).* This information is shown in Figure 11. Again, the results are classified according to the number of dendrites formed. For cases with one dendrite formation, the majority of

* This was obtained by generating a long sequence of random numbers (y_i $y_{max} = 80$) from which a number of subsets (z_i), where $z_i = y_i + k$ and k is an arbitrary integer randomly selected to represent the initial position for the various cases.

* For a dendrite with a single particle, its height would be 0.2 for the example problem.

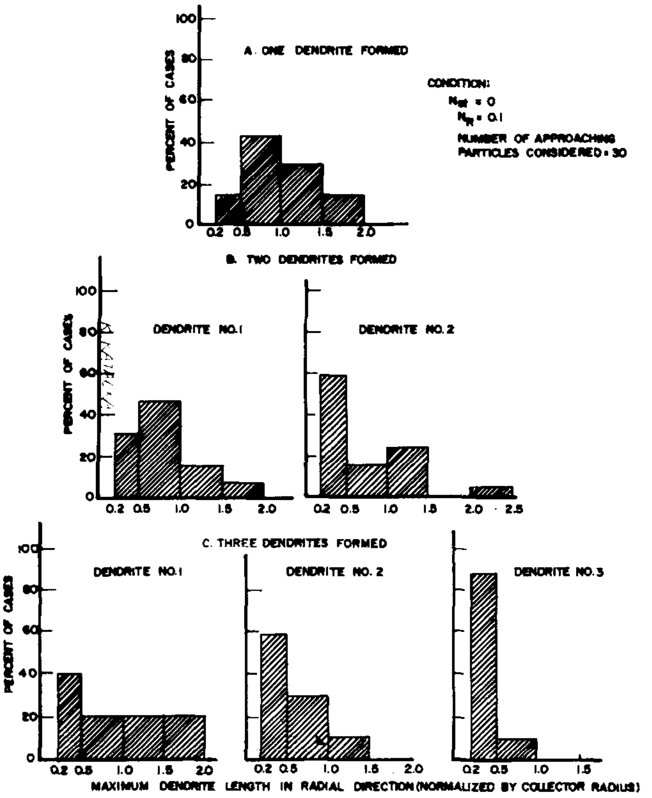

Fig. 11. Distribution by dendrite length.

these were found to have a height between 0.5 to 1.5. This value reduces to 0.5 to 1.0 for two dendrite cases. With three dendrites, the average height is less than 0.5.

Finally, to insure that particle concentration was uniform macroscopically, the distribution of the initial positions of the approaching particles is given in Table 2. For this example, the derivation from uniformity is on the average under 8%.

PROBLEMS IN SIMULATION

The principles of the simulation presented in the previous discussion are simple in concept. The actual work however may become quite tedious. This is an important consideration, since a very large number of simulations is often required because of the stochastic nature of the process. Furthermore, owing to the apparent lack of regularity in the morphology of the deposits, selection of a proper method of representing the results is not always clear-cut. The development of an efficient simulation procedure and methods of representing the simulation results are obviously very important.

Two methods have been considered for the simulation of the deposition process. At first, a graphical method was used. For example, consider Figure 5, in which the collector and particle trajectory surrounding the collector with different initial positions at the control surface are shown. This required a specification of the operating conditions, the geometry of the problem, the selection and placement of the control surface, and the designation of various points on the surface as initial positions. The trajectory was determined from the solution of the appropriate equations of particle motion (Beizaie, 1977). A sequence of random numbers was generated,* each of which corresponded to a designated position on the control surface. The outcome of each

* A brief account on the generation of random numbers is given by Moshman (1967).

TABLE 2. DISTRIBUTION OF APPROACHING PARTICLES OVER THE CONTROL SURFACE

Location	Number of particles	Expected number of particles	Actual values Expected values	Deviation
$0 \leq X > 0.1$	211	187.5	1.125	0.125
$0.1 \leq X < 0.2$	200	187.5	1.067	0.067
$0.2 \leq X < 0.3$	178	187.5	0.945	−0.055
$0.3 \leq X < 0.4$	182	187.5	0.971	−0.029
$0.4 \leq X < 0.5$	208	187.5	1.109	0.109
$0.5 \leq X < 0.6$	173	187.5	0.922	−0.078
$0.6 \leq X < 0.7$	190	187.5	1.013	0.013
$0.7 \leq X < 0.8$	158	187.5	0.843	−0.157
Total	1 500	1 500.0	avg. (absolute value)	0.079

approaching particle was determined by following the particle's trajectory. Particles making contact with either the collector or deposited particles were determined visually, and deposited particles were shown on the figure accordingly. With the presence of the deposited particle, a new limiting trajectory of the collection system could be located from which the collection efficiency could be calculated. The method is simple to apply; yet its disadvantages of excessive time requirement and errors associated with visual determination make it impractical to use for large numbers of simulations.

A more efficient method based on digital computation may be developed. First, particle trajectories corresponding to designated initial positions on the control surface are calculated, and the numerical data representing the trajectories are fitted empirically into mathematical equations. Two kinds of expressions can be used. For example, a trajectory can be expressed in a parametric form; that is

$$r = f_1(t, \overline{y_i}) \tag{7a}$$

$$\theta = f_2(t, \overline{y_i}) \tag{7b}$$

where $\overline{y_i}$ is the initial position of the trajectory on the control surface. Another form of expression assumes that

$$r = \Sigma b_i \theta^i \tag{8}$$

where the coefficients b_i's are dependent upon the initial position $\overline{y_i}$. The number of terms used in the polynomial varies from trajectory to trajectory. For the few cases considered so far, a third-order polynomial was found to be adequate. In making this empirical fit of the trajectories, it is only necessary to consider the segment close to the collector (For the example problems considered, this was taken to be the part of trajectory with $r \leq 3$.

To carry out the simulation, the trajectory with an initial position corresponding to the first member of the random number sequence will be considered first. Depending upon whether or not the trajectory traverses through a point $r = 1 + N_R$, the particles will deposit or escape. In the event that the particle escapes, a second trajectory with initial position corresponding to the second member of the random number sequence will be examined. This process will be continued until a deposition is found, and the position of the particle deposited $\overline{r_1}(r_1, \theta_1)$ will be recorded.

The next particle considered following the deposition of the first particle will have three possibilities: escape, deposition on the collector, or deposition on the first deposited particle. To determine the outcome of this particle, a sequence of points along its trajectory $[P_o(r_{po}\theta_{po}), P_1(r_{p1}\theta_{p1}) \ldots P_i(r_{pi}, \theta_{pi}) \ldots P_N(r_{pN}, \theta_{pN})]$ will be selected, and the distance between P_i and $\bar{r}_1$ and the radial distance r_{pi} are examined to determine whether either of the following conditions are satisfied.

$$r_{pi} \leq 1 + N_R \tag{9}$$

$$\frac{2}{P_i\bar{r}_1} < r_{pi}^2 + r_1^2 - 2r_{pi}r_1 \cos(\theta_{po} - \theta_1) \tag{10}$$

The requirement of Equation (9) implies deposition on the collector and that of Equation (10) indicates attachment to the previous deposited particle. In the case when either one of the inequalities is observed, the exact position of the particle upon deposition can be located by moving the particle to a point slightly upstream from P_i such that the equality condition is met. This can be done by trial and error. The deposition position is recorded as $\bar{r}_2(r_2, \theta_2)$.

The location of the initial point P_o, (r_{po}, θ_{po}) is given as

$$r_{po} = 1 + 3N_R^{*} \tag{11}$$

and θ_{po} can be found from either Equation (7*b*) or (8). The other points are selected in such a way that

$$\theta_{pi} = \theta_{pi-1} - \Delta\theta \tag{12}$$

when $\Delta\theta$ is a small angle. The selection of $\Delta\theta$ is largely a matter of judgment, and its magnitude varies from problem to problem. As soon as the angular position of P_i passes $\pi/2$ without satisfying either conditions [that is, Equations (9) and (10)], it means that the particle under consideration escapes collection.

In general, assume that M particles have been considered, among which m particles are captured with deposition positions recorded: $\bar{r}$, $(r_1, \theta_1)\bar{r}_2(r_2, {}_2\theta) \ldots \bar{r}_m(r_m, \theta_m)$. To determine the outcome of the next particle, a sequence of points $(P_o, P_1, \ldots P_N)$ are selected such that

$$r_{po} = 1 + (2m + 1)N_R \tag{13}$$

$$\theta_{pj} = \theta_{pj-1} - \Delta\theta \tag{14}$$

The criteria of deposition are

$r_{pj} \leq 1 + N_R$ (particle deposits on collector)

$$\overline{P_jr_i^2} = r_{pj}^2 + r_i^2 - 2r_{pj}r_i \cos(\theta_{pj} - \theta_i) \quad i = 1, 2, \ldots M$$

(particle deposits on the deposited particle)

In the event of deposition, and assuming that the inequality conditions are observed, the exact position can be found by trial and error as before (that is, by moving the particle slightly to the left of P_j). The particle under consideration escapes from collection if the angular position of P_j, θ_i passes through a $\pi/2$ without having either condition satisfied.

This second method is still under development. Preliminary results obtained so far have been rather encouraging. A few examples have been worked out by both methods, and the results were found identical.

Besides the need to develop more efficient simulation procedures, there are a number of problems which require careful attention because of their effects on the accuracy of the results of the simulation. A brief discussion of these problems is given below.

* This condition corresponds to the furthermost location that the particle under consideration may be deposited.

1. Finite number of designated initial positions on the control surface. The method used in representing the random behavior of the approaching particles is to divide the control surface with grid points of equal distance. In fact, these grid points are the only points through which approaching particles may pass through the control surface. This obviously is an approximation, and one would expect that for a desired accuracy a minimum number of points has to be used.

For a given control surface, if the number of points used are too small, it can be seen easily that the results obtained will depend upon the locations of these points. For example, in the example problem presented in Figure 5, if only two points are selected, such as points 0 and X_o, particles originating at point 0 would form a straight particle dendrite extending away from the stagnation point, and particles beginning at X_o would escape and the results become predetermined. Different but similarly absurd situations would arise with other kinds of selections.

From the physical point of view, it can be seen that the distance between the adjacent points should be less than the size of the particle. The quantitative information however is lacking. This problem can be solved empirically by carrying out simulation for the same problem using different grid point systems and comparing the results with one another. Such a study is being carried out by the authors presently.

2. The effect of deposited particles on the flow field around the collector. We have assumed in the example problem that deposition of particles on the collector has insignificant effects on the flow field around the collector. This certainly is not true if the extent of deposition is large or in the immediate neighborhood of the deposited particles. A rigorous solution of the flow problem around a collector with deposited solid particles would not only be difficult but impractical because of the almost infinite varieties of deposit patterns which may arise. It would, however, be useful to consider some simple geometry of deposits, such as dendrites with one or two particles, and to obtain the corresponding flow solutions. Such information can be used as a basis to assess the possible effect of particle deposits in flow field on pressure drop, the drag forces acting on the deposits, etc.

3. The order of particle deposition. In simulating the deposition process as described above, the outcome of each approaching particle is determined sequentially, that is, in the order of its arrival at the control surface. Since the deposition of a given particle is determined, to a certain degree, by the state of deposition on the collector at the time of consideration, the validity of the procedure is guaranteed if the order of particle deposits does not violate the order of particle arrivals at the control surface. In other words, consider two approaching particles, both of which will be captured. The deposition of the particle which arrives at the control surface earlier procedes that of the particle arriving later.*

A calculation of the time required to travel from the control surface to the vicinity of the collection is made for a particular case and is shown in Figure 12. The time requirement is shown to be a function of the initial position of the particle at the control surface and the distance within the collector it has to reach. It would appear that if the two approaching particles in question are situated

* An example of the possible effect of this reversible order of deposition can be seen as follows. Consider two approaching particles departing from the control surface consecutively. Based on the degree of deposition of the collector existing at the departure time of the first particle, the first particle would escape. However, if the second particle arrives at the collection first and becomes deposited at the proper location, it may capture the first particle.

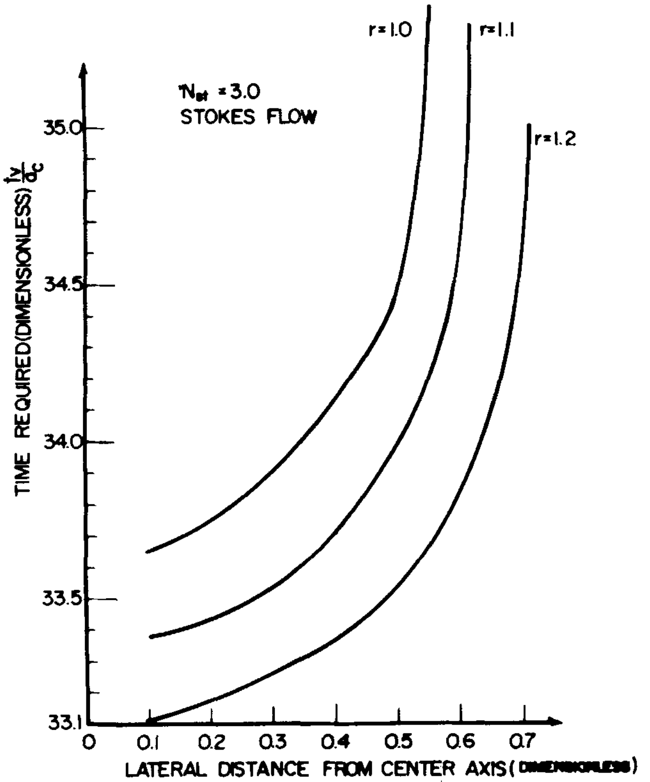

Fig. 12. Time required for particles to travel from an upstream position ($z = -30$) to a point near the collector.

far apart on the control surface, their order of deposition may not agree with that of their arrival. However, because of their being far apart, the possible effect of the deposition of the late starting particles on the earlier starting particles would be nil. An unequivocal answer to this question, however, deserves further attention.

POTENTIAL APPLICATION

Although the method of simulation described above is still in its developmental stages, and the results obtained so far are preliminary in nature, it is obvious that the method proposed in this work has significant potential in the study of deposition phenomenon. Perhaps the most important feature of this method is its ability to describe the deposition process in a detailed step-by-step manner. The amount of particle collection as well as the morphology of the deposits are given as functions of time (or the number of particles considered). The ambiguity on the use of the clean collector efficiency is removed. As a result, the difficult problem of determining the collection efficiency as a function of the extent of deposition is solved rather simply with the present method.

The knowledge of the geometry and nature of the deposits obtained from the simulation method is particularly useful on two accounts. First, the information on deposit morphology enables a more rational and a more basic study of the pressure drop built up across a filter bed during filtration. This problem has long been recognized as being practically important. The major difficulty with which previous investigators have been confronted in the study of pressure drop increase is a lack of comprehension of the underlying physical process. The picture of deposition presented by this method can be used as a basis to calculate or correlate data of pressure drop increase. Second, the morphological information about the particle deposits and about their size and arrangement can be used in estimating the force required to break off the particle dendrite as well as the optimal way of bringing about its break off. Such information is of fundamental importance to the study of filter cleaning.

Another area of study in which the present method can be fruitfully applied is the investigation of the effect of the polydispersity of particles on particle collection. For this application, in addition to randomly determining the positions of approaching particles at the control surface, the size of the particle has to be specified. This can be done in such a way that the probability of an approaching particle to be of particular size is proportional to the concentration of particle of the size in the suspension. The collection efficiency can be determined in a clear-cut way without the necessity of resorting to the use of average size. The information on deposit structure would be useful in determining the effect of extremely small or large particles on the porosity of deposited matters.

DISCUSSION

The deposition theory presented here is based on two concepts: the shadow effect of deposited particles and the singular and random behavior of approaching particles. It is interesting to note that the random behavior concept has been used previously in the study of flocculation (Hutchison and Sutherland, 1965). The major difference between the earlier flocculation investigation and the present deposition study is that in deposition, individual particles follow particle trajectories which are determined by the flow field around the collector and the operating forces. For flocculation work, particle motions were assumed to be rectilinear. Hutchison and Sutherland's theory concluded that the flocs formed have the characterization of an open structured random solid and are different from those of close packed ones. If the same conclusion holds true for deposits in a deposition process, this difference in structure must be included in the study of pressure drop increase across a filter bed due to particle accumulation.

In an earlier study, Payatakes and Tien (1976) and Payatakes (1977) proposed a theory for the formation and growth of particle dendrites on a single cylindrical collector. This earlier theory considers the kinetics of particle dendrite growth as function of the angular position of deposition on the collector. The present theory, because of its incorporation of the random behavior of approaching particles, removes the need for this assumption. The positions of deposition and the morphology, in general, are the answers to be provided by the theory. In this sense, the present theory gives a more complete account of the particle deposition process. However, since in the earlier stage of deposition, the number of dendrites formed as well as their sizes are small, the shadow effect is therefore not as important as in the later stage. Thus, the average collection efficiencies and dendrite size distributions predicted by the Payatakes-Tien model are expected to be in agreement with the present theory.

The application of the concept and the simulation technique are not limited to filtration studies only. The ice accretion problem in which the impaction and deposition of airborne small droplets of subcooled water on a cold surface results in frost formation has been a subject of intense interest. The idea and concept developed here, together with heat transfer considerations, should provide a useful framework for its study.

NOTATION

a_c, a_p = collector radius and particle radius
A = control surface area
b_i = coefficients of Equation (8)
c = particle concentration
C = Cunningham correction factor
E = efficiency defined by Equation (4)
f_1, f_2 = parametric functions representing particle trajectory
i, j = indexes of sequences
M = total number of particles considered
m = total number of particles collected among M particles considered
N_R = relative size parameter defined as the ratio of particle radius to collection radius a_p/a_c
N_{St} = Stokes number defined as $2C\rho_p V a_p^2/9\mu a_c$
N = number of particles passing through the control surface
r = radial distance
$\bar{r}_i$ = position of i^{th} deposited particle
Δt = time interval
t = time
V = approaching velocity
W = deposition rate over a collector
X = lateral position from the center axis
X_o = maximum lateral distance from the center axis on the control surface
X_{cr} = distance between the limiting trajectory and central axis upstream
$(X_{cr})_o$ = values of X_{cr} corresponding to a clean collector
η = collection efficiency
θ = angular position
μ = viscosity of fluid media
ρ_p = density of particle

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Experimental Study of Eddy Diffusion Model for Heated Turbulent Free Jets

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Prandtl's eddy diffusion model was successfully extended to turbulent momentum, energy, and mass transfer in nonisothermal free jets by taking into account the effect of change in volume of traveling eddies. The present model was experimentally confirmed for the high temperature, low velocity, turbulent free jets produced by combustion of methane gas.

SCOPE

Problems of heat and mass transfer in impinging jets have recently led to a need for information that will permit accurate prediction of the temperature, velocity, and concentration fields at various temperature levels. From an engineering viewpoint, these problems become very important, especially when a nonisothermal free jet

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